Dynamic Navigation Aspects of an Automated Medical Wheelchair

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***Abstract*— This paper deals with several aspects of a medical wheelchair. There is a mechanical three-wheels type structure that contains a single simultaneous drive and steering wheel and two passive wheels mounted on the same rear axle. The direct kinematic model and the inverse kinematic model are first developed. Then, dynamic aspects of the medical wheelchair navigation on a required trajectory are presented and analyzed. This is constituted by the succession of different types of sections, linear or curvilinear. It is intended to develop driving laws that do not introduce sudden step variations to changing sections. In this respect, the paper presents two driving solutions on complex trajectories: driving strategy through direction and speed driving strategy, respectively. For each, aspects that are more particular are considered.**

***Keywords*— medical wheelchair, modeling, navigation**

I. Introduction

There are currently a wide variety of types of wheelchair for the disabled persons, differing by propulsion method, mechanisms of control, and technology used [1], [2].

A manual self-propelled wheelchair is a non-powered system (Fig. 1). Usually it has 3-4 wheels and it can be handled by patients who do not have problems in upper limbs. The larger rear wheels usually have push-rims of slightly smaller diameter projecting just beyond the tyre; these allow the user to manouevre the chair by pushing on them without requiring them to grasp the tyres.

Fig. 1. A manual self-propelled medical wheelchair.

An electric-powered wheelchair is a wheelchair which additionally incorporates batteries and electric motors into the frame. It is controlled by either the user or an attendant, most commonly via a small joystick mounted on the armrest, or on the upper rear of the frame. For the disabled who cannot manage a manual joystick, this type of wheelchair can be equipped with an automatic control system that can ensure the movement on preset, often memorized trajectories. Certain medical requirements related to the transport of the person with disabilities can be accomplished for daily control in a particular medical destination, return to the salon, daily ride, sports and recreation (Fig. 2) etc.

Fig. 2. Electric-powered medical wheelchair for sports and recreation.

A smart or automated wheelchair is any powerchair using a control system to augment or replace user control. Its purpose is to reduce or eliminate the user's task of driving a powerchair. Usually, a smart wheelchair is controlled via a computer, it has a suite of sensors and it applies techniques in mobile robotics. Smart wheelchairs are designed for a variety of user types. Some are designed for users with cognitive impairments, such as dementia, users living with severe motor disabilities, such as cerebral palsy, or with quadriplegia. These controlled wheelchairs typically apply collision-avoidance techniques to ensure that users do not accidentally select a drive command that results in a collision and typically employ techniques from artificial intelligence, such as path-planning. Fig.3 illustrates a smart wheelchair with a 3-wheels structure (or tricycle). It contains in the front a wheel that simultaneously performs the traction function as well as the steering control of the wheelchair. Two passive wheels mounted on the same axle are located at the rear of the wheelchair. An inverted constructive solution is also possible and used.

Fig. 3. A smart wheelchair with a single motrico-driving wheel.

This work is organized like this. Section 2 introduces the notations used for modeling and sets out the equations of the direct and the inverse kinematic models. Section 3 introduces some general aspects of navigation on a trajectory composed of different types of concatenated segments, linear or curved. In Section 4, the steering strategy is developed in the direction of choice of the characteristic wheelchair control point on the imposed trajectory. It is intended to develop in the future a driving law that does not introduce sudden step variations when changing trajectory sections. Section 5 presents a second speed driving strategy that takes into account the minimum stopping distance on the medical wheelchair.

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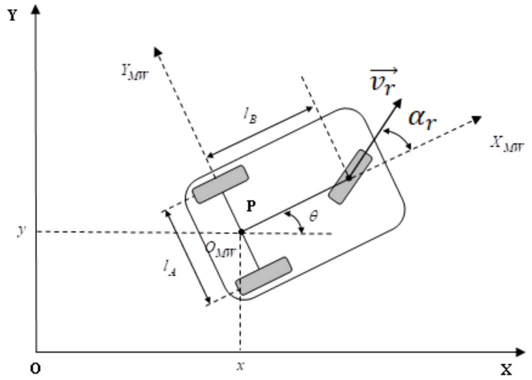


Fig. 4. The medical wheelchair with a single driving wheel.

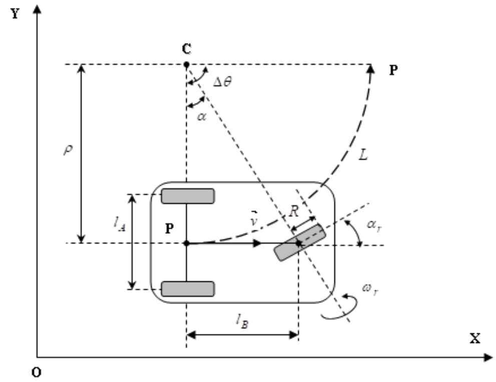


Fig. 5. The medical wheelchair, notations used for their models..

II. MEDICAL WHEELCHAIR MODEL

We will consider a medical wheelchair model with a single driving wheel, as in Fig. 4. Such a mechanical structure uses a single wheel previously displaced relative to the normal direction of movement for both functions: *the drive or advance function* (by the angular speed control) and *the steering function* (by controlling the wheel plane orientation relative to the longitudinal center line of the wheelchair). The rear axle contains two independent rotation wheels that are exclusively designed to achieve isostatic sustainability. This mechanical structure is also known as tricycle [3],[4].

If v_r represents the velocity command and α_r represents the steering command applied to the previous drive wheel, the following nonlinear kinematic equations can be determined. These equations describe the behavior of the medical wheelchair relative to the absolute reference system attached to the operating scene, where l_B represents the length of the longitudinal median axis of the wheelchair (Fig. 4):

$$\dot{x} = v_r \cdot \cos\theta \cdot \cos\alpha_r \tag{1}$$

$$\dot{y} = v_r \cdot \sin\theta \cdot \cos\alpha_r \tag{2}$$

$$\dot{\theta} = \frac{v_r}{l_B} \cdot \sin\alpha_r \tag{3}$$

The state of the robot is represented by its position and its momentary orientation in the same reference system attached to the operatin scene:

$$\bar{x} = (x, y, \theta)^T \tag{4}$$

The presence of a single driving wheel determines the control vector as being composed of two elements: the velocity command v_r (applied by the traction subsystem of the medical wheelchair) and the angle command α_r (applied by the steering subsystem of the medical wheelchair).

$$\bar{u} = (v_r, \alpha_r)^T \tag{5}$$

Under these conditions and using the matrix notation, one can write the next equation:

$$\dot{\bar{x}} = F(\bar{x}, \bar{u}) = \begin{pmatrix} v_r \cdot \cos\theta \cdot \cos\alpha_r \\ v_r \cdot \sin\theta \cdot \cos\alpha_r \\ \frac{v_r}{l_B} \cdot \sin\alpha_r \end{pmatrix} \tag{6}$$

where $F(\bar{x}, \bar{u})$ is the nonlinear matrix associated with the model of the medical wheelchair.

To write the following equations were used the notations introduced by the Fig. 5. The angle of rotation of the driving wheel of the medical wheelchair is given by:

$$\alpha_r = arctg\frac{l_B}{\rho} \tag{7}$$

The linear speed of the medical wheelchair (associated with its characteristic point *P*) can be determined on the basis of the angular velocity ω_r of the driving wheel:

$$v = R \cdot \omega_r \cdot \cos\alpha_r \tag{8}$$

The rotation speed of the robot around the center of the momentary trajectory will be:

$$\dot{\theta} = \frac{R}{l_B} \cdot \omega_r \cdot \sin\alpha_r \tag{9}$$

Equations (8) and (9) constitute *the direct kinematic model* of the tricycle medical wheelchair.

Knowing the angular speed of the drive wheel ω_r and the angle of rotation of this wheel α_r, it is possible to determine the linear and the angular speed of the tricycle wheelchair at its characteristic point *P*. The center angle described by the medical wheelchair can be determined as follows:

$$\rho = \frac{v}{\dot{\theta}} = l_B \cdot ctg\alpha, \qquad \alpha = arctg\left(\frac{l_B}{\rho}\right) \tag{10}$$

By rewriting the equation (8) we obtain:

$$\omega_r = \frac{v}{R \cdot \cos\alpha} \tag{11}$$

and from equation (10) it follows:

$$\alpha_r = arctg\left(\frac{\dot{\theta}}{l_B \cdot v}\right) \tag{12}$$

Equations (11) and (12) constitute *the inverse kinematic model* of the tricycle medical wheelchair. These equations allow both the determination of the angular velocity ω_r and the orientation α_r of the drive wheel of the wheelchair, elements necessary to achieve an imposed trajectory characterized by linear velocity v and angular orientation θ.

III. NAVIGATION ASPECTS

A section of the imposed trajectory can be defined by the following elements:

- coordinates of the starting point of the section in the absolute reference to the operating scene
- orientation of the section in the absolute benchmark
- the length L of the section
- curvature k of the section ($k = 1/\rho$, where ρ is the radius of the section)
- the direction of travel on the section, i.e. forward or backward
- the characteristic point or the control point of the medical weelchair relative to the section

The following sign rule shall be used to indicate the direction of rotation on a trajectory: the turn in the trigonometric sense is positive and the turn in the clockwise direction is negative. The curvature of the segment is null for the straight segment and nonzero rest (in particular it is constant for a circle arc). The M point of the wheelchair control relative to the current section of the trajectory belongs to its longitudinal axis. In the general case, it can occupy any position, being defined by the distance measured relative to the middle P of the rear axle as in Fig. 7:

- $l_C = 0$ signifies its placement at this point
- $l_C = l_B$ marks the placement in the center of the driving wheel

The dynamics of the tricycle medical wheelchair on the imposed trajectory is restricted by its mechanical, cinematic and dynamic characteristics [7], [8], and can be characterized by the following maximum values:

- speed of evolution per section $v_{\max}$
- acceleration per section $a_{a,\max}$
- deceleration per section $a_{d,\max}$
- stopping distance per section $d_{\max}$

The maximum stopping distance on a section is related to the maximum speed and the maximum deceleration by the next equation:

$$d_{\max} = \frac{v_{\max}^2}{2 \cdot a_{d,\max}} \tag{13}$$

For the assessment of the other dynamic parameters, the geometric shape of the trajectory section should be considered, as follows:

- If the section is a straight segment, the choice of the characteristic point is irrelevant because all the points of the longitudinal axis are subject to the same mode. The maximum speed, acceleration and deceleration are determined by the choice of the engine that will provide the wheelchair with its motoring function.
- If the section has a certain curvature k, the higher the curvature value, the velocity must be decreased in order to limit the effect of the centrifugal force that may affect the stability of the transported person or even wheel slip on the ground. Thus, if the centrifugal acceleration is limited to the value $a_{cf\max}$ and the curvature k of the section is known, the maximum speed of the characteristic point is given by:

$$v_{\max} = \sqrt{\frac{a_{cf\max}}{|k|}} \tag{14}$$

IV. DRIVING STRATEGY THROUGH DIRECTION

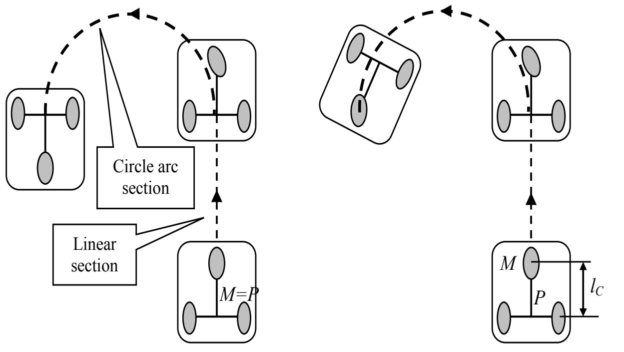
In this type of driving, the problem is to establish the law of evolution of the prescribed value for the angle of rotation α^* of the driving wheel. Depending on the position chosen along the longitudinal axis for the control point M in relation to the current section of the trajectory, two cases can be defined:

Case A. The control point M coincides with the characteristic P point of the medical wheelchair, that is $l_C = 0$. Choosing the control point even at the characteristic point (Fig. 6.a) guarantees the orientation at the end of the trajectory, which may be of particular importance in certain applications. However, there is a need for the instantaneous modification of the direction of engagement of arc-arc or linear-arc segments of the trajectory with a value:

$$\alpha^* = arctg\, k \tag{15}$$

But this is practically not possible due to the non-zero response time of the steering subsystem. To compensate for errors in tracking the reference trajectory, it is necessary to adopt a correction method.

Case B. The control point M does not coincide with the characteristic point P, so $l_C \neq 0$. The behavior of the wheelchair will be that of Fig. 6.b, presenting the inconvenience of not guiding at the end of the trajectory. This may be an acceptable situation when the targeting parameter is not critical, such as the case of the final positioning within a particular area for serving the disabled.



a. Case $l_C = 0$ $(M \equiv P)$ b. Case $l_C \neq 0$ $(M \neq P)$

Fig. 6. Driving by direction depending on the choice of control point.

Therefore, it is important to develop a driving control law that does not introduce sudden step variations to changing sections. The basic principle of the next theoretical development (Fig. 7) consists in the insertion at the control point of a fictitious wheel, replacing the previous wheel-drive wheel, to ensure the same behavior for the wheelchair.

Using the notations introduced in Fig. 8, if β_M is the fictitious wheel orientation, its instantaneous velocities can be expressed according to the speed v^* of the actual driving wheel by the relation:

$$tg\beta_M = \frac{v_{M\,y}}{v_{M\,x}} = \frac{l_C}{l_B} \cdot \frac{v_y^*}{v_x^*} = \frac{l_C}{l_B} \cdot tg\alpha^* \qquad (16)$$

The dynamic equations that can be determined are as follows:

$$\frac{d\theta}{dt} = \frac{v_{M\,y}}{l_C} \qquad (17)$$

If λ is the curvilinear abscissa of the trajectory imposed at the characteristic point M, size defined by $\lambda = v \cdot \Delta t$, then:

$$\frac{d\lambda}{dt} = v_M = \sqrt{v_{M\,x}^2 + v_{M\,y}^2} = v_{M\,y} \cdot \sqrt{1 + \frac{1}{tg^2 \beta_M}} =$$

$$= v_{M\,y} \cdot \sqrt{1 + \frac{1}{\left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2}} \qquad (18)$$

By processing the relationship (18), the expression for speed component $v_{M\,y}$ can be obtained immediately as:

$$v_{M\,y} = \frac{d\lambda}{dt} \cdot \frac{\frac{l_C}{l_B} \cdot tg\alpha^*}{\sqrt{1 + \left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2}} \qquad (19)$$

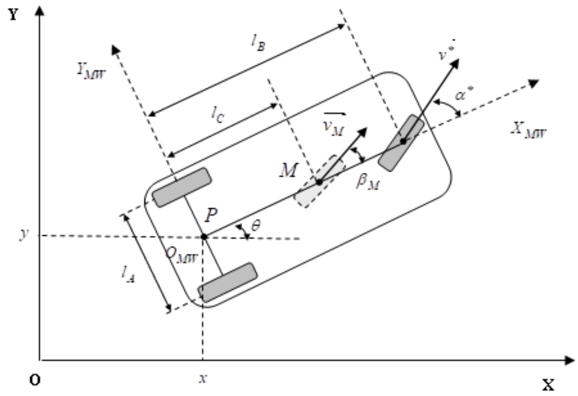


Fig. 7. The wheelchair with motrico-steering wheel and fictional wheel.

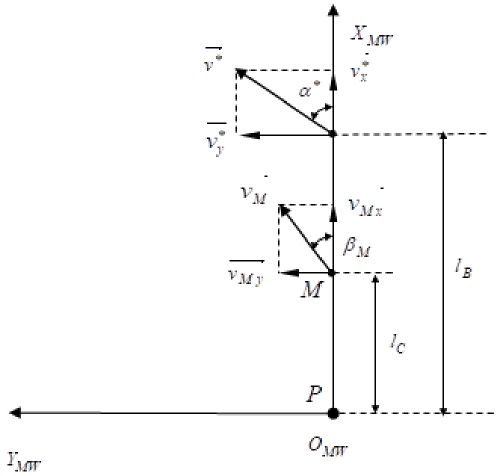


Fig. 8. Speed components for the fictitious wheel structure.

and from the relations (17) and (19) it results the next equation:

$$\frac{d\theta}{d\lambda} = \frac{1}{l_B} \cdot \frac{tg\alpha^*}{\sqrt{1 + \left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2}} \qquad (20)$$

The imposed value for the steering angle of the drive wheel can now be estimated by the relationship:

$$\alpha^* = \beta_M + \theta \qquad (21)$$

- If the trajectory section is a straight segment type:

$$\alpha^* = 0,\ \beta_M = 0 \rightarrow \frac{d\theta}{dt} = 0,\ \frac{d(\beta_M + \theta)}{d\lambda} = 0 \qquad (22)$$

- If the trajectory section is an circle arc of radius *R*:

$$\frac{d(\beta_M + \theta)}{d\lambda} = \pm \frac{1}{R} \qquad (23)$$

In these two last cases, if the curvature of the section, denoted by k and defined as the inversion of the radius of the section ($k > 0$ for left or trigonometric sense), is obtained the next equations:

$$k = \frac{d(\beta_M + \theta)}{d\lambda} = \frac{d\beta_M}{d\lambda} + \frac{d\theta}{d\lambda} \tag{24}$$

where:

$$\beta_M = arctg\left(\frac{l_C}{l_B} \cdot tg\alpha^*\right) \tag{25}$$

$$\frac{d\beta_M}{d\lambda} = \frac{d\beta_M}{d\alpha^*} \cdot \frac{d\alpha^*}{d\lambda} = \frac{1}{1 + \left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2} \cdot \frac{d\alpha^*}{d\lambda} \tag{26}$$

that is, we will get the following relationship:

$$k = \frac{1}{l_B} \cdot \frac{tg\alpha^*}{\sqrt{1 + \left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2}} + \frac{\frac{l_C}{l_B} \cdot \left(1 + tg^2\alpha^*\right)}{1 + \left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2} \cdot \frac{d\alpha^*}{d\lambda} \tag{27}$$

Finally, the driving strategy law through direction to tracking a curved k-section of imposed trajectory according to the position of the characteristic point is:

$$\frac{d\alpha^*}{d\lambda} = A(\alpha^*) \cdot k + B(\alpha^*) \tag{28}$$

where:

$$A(\alpha^*) = \frac{1 + \left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2}{\frac{l_C}{l_B} \cdot \left(1 + tg^2\alpha^*\right)} \quad B(\alpha^*) = \frac{tg\alpha^* \cdot \sqrt{1 + \left(\frac{l_C}{l_B} \cdot tg\alpha^*\right)^2}}{l_C \cdot \left(1 + tg^2\alpha^*\right)} \tag{29}$$

By comparing the two previously established types of driving strategy through direction on the required trajectory, it can be specified that it is possible to use them alternatively. The selection is depending on the purpose pursued on the current section of the trayectory.

Thus, if the final orientation must be accurate, the case presented in Fig. 6.a, it is necessary to adopt a control strategy that takes into account the characteristic point placed in the middle of the posterior axis.

If the orientation of the tricycle medical wheelchair is not an essential parameter, the case presented in Fig. 6.b in which the operating scene is not severely restricted or the final positioning is of a zonal type, the steering strategy in the direction for a characteristic point conveniently placed on the center line can be adopted.

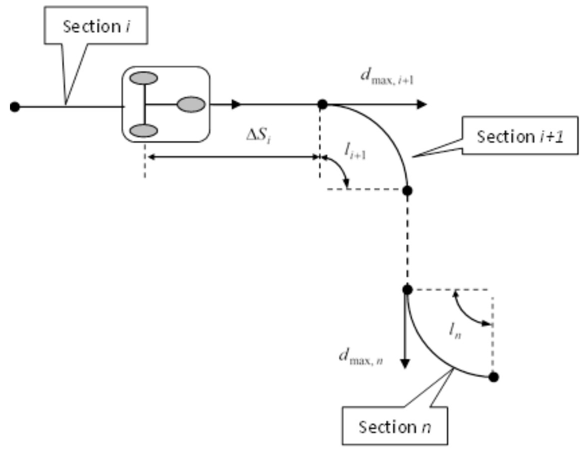


Fig. 9. Minimum stopping distance method.

The selection of the strategy appropriate to the functional purpose can be achieved by the decision of the higher hierarchical level of the automated medical wheelchair [5], depending on the external sensory information or on internal data memorized by the control system.

V. Speed Driving Strategy

We consider a trajectory formed by n trajectory sections (or trajectory segments) with respective lengths $l_1, l_2, \dots l_n$ and a medical automated wheelchair located on the inte-mediate section i.

According to the relation (13), for each section is defined a stopping distance, respective $d_{\max 1}$, $d_{\max 2}$,..., $d_{\max n}$. Given the required profile of the guidance trajectory, it is a question of determining the prescribed speed for the traction subsystem of the automated medical wheelchair. This value can be estimated using a method which follows the minimum stopping distance.

If ΔS_i represents the distance between the characteristic point of the automated tricycle medical wheelchair and the end point of the current section i, the maximum value $d_{\max}$ of the minimum stop distance on the trajectory can be evaluated according to Fig. 9 through next relationship:

$$\begin{aligned} d_{\max} = \min \{ &(\Delta S_i + d_{\max,\, i+1}),\ (\Delta S_i + l_{i+1} + d_{\max,\, i+2}), \dots, \\ &(\Delta S_i + l_{i+1} + l_{i+2} + \dots + l_{n-1} + d_{\max,\, n}), \\ &(\Delta S_i + l_{i+1} + l_{i+2} + \dots + l_n)\} \end{aligned} \tag{30}$$

If we do not take into account the presence of a possible obstacle in the direction of deplacement of the medical wheelchair, we can determine the value of linear speed v_{stop} that allows stopping on $d_{\max}$ distance using the maximum possible and constant deceleration per the current section of the trajectory:

$$v_{stop} = \sqrt{2 \cdot |a_{d,\max}| \cdot d_{\max}} \tag{31}$$

The linear speed per the current section being limited to the value v_{max} , the prescribed speed for the automated wheelchair will be:

$$v^* = \min\{v_{max}, v_{stop}\} \quad (32)$$

The presented method guarantees do not exceed the maximum speed on the current section, allows the initial input speeds on each section of the trajectory and stopping at the end point of the last section of the trajectory.

If the wheelchair's sensory self-sensing system detects the presence of an unforeseen obstacle in the critical security area, then automatically the prescribed speed becomes null. Let's write with $d_{obst.j}$, where $j = \overline{1,r}$, the security distances provided at one time by the r sensors that control the front of the automated medical wheechair. The stop distance in the presence of the unforeseen obstacle can be defined as:

$$d_{obst.} = \min\{d_{obst.j}, j = \overline{1,r}\} \quad (33)$$

We can now integrate this distance in the minimum stopping distance calculation as follows:

$$d_{stop} = \min\{d_{max}, d_{obst.}\} \quad (34)$$

The prescribed speed for the automated medical wheelchair will be similarly reevaluated as follows:

$$v_{stop} = \sqrt{2 \cdot |a_{d,max}| \cdot d_{stop}} \quad (35)$$

$$v^* = \min\{v_{max}, v_{stop}\} \quad (36)$$

If $d_{max} < d_{obst.}$, then the prescribed speed v^* is adjusted so that the automated medical wheelchair stops at the end of the trajectory, and if $d_{max} > d_{obst.}$, then v^* is calculated so that the automated medical wheelchair stops at the distance $d_{obst.}$ provided by the sensors.

Relationships (32) and (36) lead to the conclusion that the regulation of the speed and the acceleration of the automated medical wheelchair must be approached [6] under the conditions of an algorithm with imposed dynamic limitations.

VI. Conclusions

This paper deals with several aspects of an automated medical wheelchair, a mechanical tricycle type structure that contains a single drive and steering wheel placed in the front and two passive wheels, nonactuated, mounted on the same rear axle. For this constructive solution, the direct kinematic model and the reverse kinematic model are first developed.

Then there are presented and analyzed dynamic aspects of the medical wheelchair navigation on a required trajectory. This imposed trajectory is constituted by the succession of different types of sections (or segments), as linear or curvilinear. It is intended to develop driving laws that do not introduce sudden step variations when sections are changed. In this respect, the paper presents two driving solutions on complex trajectories: driving strategy through direction and speed driving strategy, respectively. For each of them, more particular aspects are considered.

In the driving strategy through direction was established two types of control. It is possible to use them alternatively, depending on the purpose pursued on the current section. The selection of the strategy appropriate to the functional purpose can be achieved by the decision of the higher hierarchical level of the automated medical wheelchair, depending on the external sensory information or on internal data memorized by the control system.

The second method, speed driving strategy, leads to the conclusion that the regulation of the linear speed and the acceleration of the automated wheelchair must be approached under the conditions of an algorithm with imposed dynamic limitations.

The content of this paper mainly includes the theoretical approaches and the mathematical foundations that will underpin our future experimental developments for the design and construction of an automated medical wheelchair in the tricycle-type mechanical version.

Acknowledgment

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